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Delay-dependent robust stability analysis for neutral systems with discrete and distributed delays

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***Abstract*— This paper addresses the robust stability analysis of uncertain neutral systems with discrete and distributed delays. An innovative approach is developed for the derivation of delay-dependent stability criteria. Like other methods, the approach reported here also employes the technique of linear matrix inequalities. However, unlike the existing methods that are popularly used in recent publications, our approach employes neither free weighing matrices nor model transformation, and is thus much simpler. With the proposed approach, delay-dependent robust stability criteria are derived for the systems under consideration, leading to significant improvement in the allowable delay bound. Numerical simulations are given to demonstrate the effectiveness of the proposed approach.**

Keywords: Neutral system; Robust stability; Uncertain systems; Delay-dependent stability; Distributed delay

I. INTRODUCTION

Stability analysis of neutral systems with discrete and distributed delay has received increasing attention in recent years, and has been one of the interesting topics in control systems. Practical examples of neutral delay-differential systems include aircraft stabilization, chemical engineering processes, distributed networks, neural networks, nuclear reactors, and population dynamics [1], [2]. Introductions to delay-dependent stability results for neutral systems with distributed delays can be found in [3], [4], [5], [6], [7] and references therein.

An important topic in robust stability analysis for dynamical systems with delay is how to increase the allowable delay bound without loss of the system stability. Several approaches have been developed in the last few years to reduce the conservation of delay-dependent stability conditions for neutral systems with distributed delay. Gu [4] applied the discretized Lyapunov functional approach in delay-dependent stability analysis for systems with distributed delays. Zheng and Frank [8] employed a first-order transformation method in their study of the stability and stabilization of systems with distributed delays. However, their results are conservative due to the introduction of additional synamics from the first-order transformation. Later, Yue, Won and Kwon [7] developed a parameterized model transformation method to deal with similar stability problems. The descriptor model transformation technology has also been developed, originally by Fridman and colleagues [6], [9], later by Han [5], and most recently by Chen and Zheng [3].

In the above mentioned approaches, the descriptor model transformation technology developed in [6], [9], [5], [3] is less conservative than the first-order transformation method in [8] and the parameterized model transformation in [7]. In the early development of the descriptor model transformation technology [6], [9], [5], the distributed delay terms were treated as perturbations and the discrete delay terms were rewritten via integration. In the most recent advances in the technology by Chen and Zheng [3], both the discrete delay terms and distributed delay terms were rewritten via the descriptor model transformation.

It is noticed that the work by Chen and Zheng [3] employes Moon's inequalities [10], [11], which are a source of conservatism. This implies that there is a potential to further improve the results of Chen and Zheng. Recently, much effort has been devoted to the free weighting matrix method, which introduces free weighting matrices to express the relations between the terms in the Leibniz-Newton formula [12], [13], [14], [15], [16]. The free weighting matrix method can be viewed as an extension of the descriptor system method of Fridman [9], [17] by adding some null summing terms to the derivative of the Lyapunov function. Moreover, this method uses a long list of parameters, which need to be tuned when the method is applied to the delay dependent controller synthesis problem.

We have understood that model transformation is a main source of conservatism [18], and the non-tighter bounding techniques for cross-product terms, e.g., Moon's inequalities [10], [11], are also a source of conservatism. It is yet to be systematically investigated how to avoid these sources of conservatism for improved stability conditions for dynamical systems with discrete and distributed delays. This motivates the research of this work.

This paper develops an innovative method to deal with the problem of the delay-dependent stability of uncertainty neutral systems with discrete and distributed delays. An augmented Lyapunov function is employed together with a tighter bounding technology for cross terms for conservatism reduction. Different from the most recent advances in this area [3], [19], [20], our approach reported in this paper employes neither model transformation nor free weighting matrices in the derivation of the stability conditions. With our developed approach[21], significant improvement has been achieved in the stability performance. Two numerical examples are provided to demonstrate the effectiveness of the developed approach.

Notation: Throughout the paper, $\mathbb{N}$ stands for positive integers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). For an arbitrarily matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

II. Problem Statement

Consider the following uncertain neutral system with discrete and distributed delays

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + A_1(t)x(t-h_1) && (1)\\ &\quad +C(t)\dot{x}(t-h_1) + A_2(t)\int_{t-h_2}^{t} x(s)ds && (2)\\ x(t) &= \phi(t), t \in [-\max(h_1,h_2), 0] && (3) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector. It is assumed that the time delay $h_1 \geq 0$, $h_2 \geq 0$ are known. $\varphi(t)$ is the initial condition of the system.$A(t)$, $A_1(t)$, $C(t)$ and $A_2(t)$ are matrix functions with time-varying uncertainties,

$$\begin{aligned} A(t) &= A + \Delta A, A_1(t) = A_1 + \Delta A_1, && (4)\\ C(t) &= C + \Delta C, A_2(t) = A_2 + \Delta A_2. && (5) \end{aligned}$$

where A, A_1, C and A_2 are constant matrices with appropriate dimensions. ΔA, ΔA_1, ΔC and ΔA_2 denote the parameter uncertainties satisfying the following conditions:

$$\begin{bmatrix} \Delta A & \Delta A_1 & \Delta C & \Delta A_2 \end{bmatrix} = MF(t)\begin{bmatrix} E_0 & E_1 & E_c & E_2 \end{bmatrix} \quad (6)$$

where M, E_0, E_1, E_2 and E_c are constant matrices with appropriate dimensions and $F(t)$ is an unknown time-varying matrix, which is Lebesque measurable in t and satisfies $F^T(t)F(t) \leq I$. When $F(t) = 0$, system (1)–(3) is referred to as a nominal linear neutral system.

In the following, we will develop some practically computable criteria for the analysis of the neutral system (1)–(3). The following lemma prevents a tighter bound to deal with cross terms and is useful in deriving the criteria.

Lemma 1: [21] For any constant matrices $Q_{11}, Q_{22}, Q_{12} \in \mathbb{R}^{n\times n}$, $Q_{11} \geq 0$, $Q_{22} > 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$, scalars $h_2 \geq 0$, $h_1 \geq 0$, and vector function $\dot{x} : [-h_2, 0] \to \mathbb{R}^n$ such that the integrations in the following relations are well defined, then

$$\begin{aligned} &1) \quad -(h_2 - h_1)\int_{t-h_2}^{t-h_1} \dot{x}^T(t)Q_{22}\dot{x}(t)dt \\ &\leq \quad \eta_1^T(t)\begin{bmatrix} -Q_{22} & Q_{22} \\ Q_{22} & -Q_{22} \end{bmatrix}\eta_1(t) && (7)\\ &2) \quad -h_2\int_{t-h_2}^{t}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}dt \\ &\leq \quad \eta_2^T(t)\begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ * & -Q_{22} & Q_{12}^T \\ * & * & -Q_{11} \end{bmatrix}\eta_2(t) && (8) \end{aligned}$$

where

$$\begin{aligned} \eta_1(t) &= \begin{bmatrix} x^T(t-h_1) & x^T(t-h_2) \end{bmatrix}^T. \\ \eta_2(t) &= \begin{bmatrix} x^T(t) & x^T(t-h_2) & (\int_{t-h_2}^{t} x(t)dt)^T \end{bmatrix}^T. \end{aligned}$$

III. Main Result

The following theorem gives an improved delay-dependent stability condition for neutral systems with discrete and distributed delays.

For notational simplicity, let

$$\begin{aligned} \tilde{Q} &= \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0, \tilde{P} = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \geq 0, \\ \tilde{S} &= \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \geq 0, \tilde{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \geq 0. \quad (9) \end{aligned}$$

Theorem 1: Given scalars $\bar{h}_1 \geq 0$ and $h_2 \geq 0$, the neutral system described by (1)-(3) is asymptotically stable for any $0 \leq h_1 \leq \bar{h}_1$, if there exist matrices $R_i (i = 1, \ldots, 4)$, $\tilde{Q} \geq 0$, $\tilde{P} \geq 0$, $\tilde{S} \geq 0$ and $\tilde{Z} \geq 0$, such that the following LMIs holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \quad (10)$$

where

$$\begin{aligned} \Sigma_{11} &= \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & R_4 & A_1^T Z_{12} \\ * & * & -S_{11} & 0 & 0 & P_{12}^T A_2 \\ * & * & * & -R_1 & C^T Z_{12} & \Omega_{46} \\ * & * & * & * & \Omega_{55} & \Omega_{56} \\ * & * & * & * & * & \Omega_{66} \end{bmatrix} \\ \Sigma_{12} &= \begin{bmatrix} A^T R_1 & \bar{h}_1 A^T S_{22} & h_2 A^T Q_{22} & hA^T R_4 \\ A_1^T R_1 & \bar{h}_1 A_1^T S_{22} & h_2 A_1^T Q_{22} & hA_d^T R_4 \\ 0 & 0 & 0 & 0 \\ C^T R_1 & \bar{h}_1 C^T S_{22} & h_2 C^T Q_{22} & hC^T R_4 \\ 0 & 0 & 0 & 0 \\ A_2^T R_1 & \bar{h}_1 A_2^T S_{22} & h_2 A_2^T Q_{22} & hA_2^T S_{22} \end{bmatrix} \\ \Sigma_{22} &= diag\{-R_1, -S_{22}, -Q_{22}, -R_4\}, h = (\bar{h}_1 - h_2) \end{aligned}$$

and

$$\begin{aligned}
\Omega_{11} &= R_0A + A^TR_0 + R_2 + R_3 + P_{12} + P_{12}^T \\
&\quad + P_{11}A + A^TP_{11}^T + \bar{h}_1^2(S_{11} + S_{12}A + A^TS_{12}^T) \\
&\quad - S_{22} + h_2^2(Q_{11} + Q_{12}A + A^TQ_{12}^T) \\
&\quad - Q_{22} + Z_{12} + Z_{12}^T + Z_{11}A + A^TZ_{11}^T, \\
\Omega_{12} &= R_0A_1 - R_2 + P_{11}A_1 - P_{12} \\
&\quad + \bar{h}_1^2S_{12}A_1 + S_{22} + h_2^2Q_{12}A_1 + Z_{11}A_1, \\
\Omega_{13} &= -S_{12}^T + AP_{12} + P_{22}, \\
\Omega_{14} &= R_0C + \bar{h}_1^2S_{12}C + h_2^2Q_{12}C \\
&\quad - A^TP_{11}C - P_{12}^TC - A^TZ_{11}C - Z_{12}^TC, \\
\Omega_{15} &= Q_{22} - Z_{12}, \\
\Omega_{16} &= R_0A_2 + P_{11}A_2 + \bar{h}_1^2S_{12}A_2 + h_2^2Q_{12}A_2 \\
&\quad - Q_{12}^T + Z_{11}A_2 + A^TZ_{12} + Z_{22}, \\
\Omega_{22} &= -R_4 - S_{22}, \Omega_{23} = S_{12}^T + A_1^TP_{12} - P_{22}, \\
\Omega_{24} &= -A_1^TP_{11}C + P_{12}^TC - A_1^TZ_{11}C \\
\Omega_{55} &= -Q_{22} - R_3 - R_4, \Omega_{56} = Q_{12}^T - Z_{22}, \\
\Omega_{46} &= -C^TP_{11}A_2 - C^TZ_{12}A_2, \\
\Omega_{66} &= -Q_{11} + Z_{12}^TA_2 + A_2^TZ_{12}.
\end{aligned}$$

Proof: Construct a Lyapunov functional candidate with eight additive terms

$$V(t) = \sum_{k=1}^{8} V_k(t) \tag{11}$$

where

$$\begin{aligned}
V_1(t) &= \int_{t-h_1}^{t} \dot{x}^T(t)R_1\dot{x}(t)dt, &(12)\\
V_2(t) &= \int_{t-h_1}^{t} x^T(t)R_2x(t)dt, &(13)\\
V_3(t) &= \int_{t-h_2}^{t} x^T(t)R_3x(t)dt, &(14)\\
V_4(t) &= (h_1 - h_2)\int_{-h_1}^{-h_2}\int_{t+s}^{t} \dot{x}^T(t)R_4\dot{x}(t)dtds, &(15)
\end{aligned}$$

$$\begin{aligned}
V_5(t) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{S} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dtds, &(16)\\
V_6(t) &= \xi_1^T(t)H\tilde{P}H^T\xi_1(t) &(17)\\
V_7(t) &= h_2\int_{-h_2}^{0}\int_{t+s}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{Q} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dtds, &(18)\\
V_8(t) &= \xi_2^T(t)H\tilde{Z}H^T\xi_2(t) &(19)
\end{aligned}$$

where

$$\begin{aligned}
\xi_1^T(t) &= \begin{bmatrix} x^T(t) & x^T(t-h_1) & (\int_{t-h_1}^{t} x(t)dt)^T \end{bmatrix} \\
\xi_2^T(t) &= \begin{bmatrix} x^T(t) & x^T(t-h_1) & (\int_{t-h_2}^{t} x(t)dt)^T \end{bmatrix} \\
H &= \begin{bmatrix} I & 0 \\ -C^T & 0 \\ 0 & I \end{bmatrix}
\end{aligned}$$

and $R_i (i = 1, \ldots, 4) > 0$, $\tilde{Q} \geq 0, \tilde{P} \geq 0, \tilde{S} \geq 0$ and $\tilde{Z} \geq 0$ are to be determined. The time derivative of $V(t)$ is taken along the state trajectory (1)-(3) yielding

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-h_1)R_1\dot{x}(t-h_1) &(20)\\
\dot{V}_2(t) &= x^T(t)R_2x(t) - x^T(t-h_1)R_2x(t-h_1) &(21)\\
\dot{V}_3(t) &= x^T(t)R_3x(t) - x^T(t-h_2)R_3x(t-h_2) &(22)
\end{aligned}$$

From (7) in Lemma 1, we have

$$\begin{aligned}
\dot{V}_4(t) &= (h_1 - h_2)^2\dot{x}^T(t)R_4\dot{x}(t) \\
&\quad - (h_1 - h_2)\int_{t-h_2}^{t-h_1} \dot{x}^T(t)R_4\dot{x}(t)ds \\
&\leq (\bar{h}_1 - h_2)^2\dot{x}^T(t)R_4\dot{x}(t) \\
&\quad + \eta_1^T(t)\begin{bmatrix} -R_4 & R_4 \\ * & -R_4 \end{bmatrix}\eta_1(t) \qquad (23)
\end{aligned}$$

From (8) in Lemma 1, it follows that

$$\begin{aligned}
\dot{V}_5(t) &= h_1^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{S} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad - h_1\int_{t-h_1}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{S} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \\
&\leq \bar{h}_1^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{S} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad + \xi_1^T(t)\begin{bmatrix} -S_{22} & S_{22} & -S_{12}^T \\ * & -S_{22} & S_{12}^T \\ * & * & -S_{11} \end{bmatrix}\xi_1(t) \qquad (24)
\end{aligned}$$

The first term of the right-hand side of (24) can be expressed as

$$\begin{aligned}
&\bar{h}_1^2 \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \tilde{S} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&= \bar{h}_1^2\xi^T(t) \begin{bmatrix} I & A^T \\ 0 & A_1^T \\ 0 & 0 \\ 0 & C^T \\ 0 & 0 \\ 0 & A_2^T \end{bmatrix} \tilde{S} \begin{bmatrix} I & A^T \\ 0 & A_1^T \\ 0 & 0 \\ 0 & C^T \\ 0 & 0 \\ 0 & A_2^T \end{bmatrix}^T \xi(t) \qquad (25)
\end{aligned}$$

where

$$\xi^T(t) = \begin{bmatrix} \xi_1^T(t) & \dot{x}^T(t-h_1) & x^T(t-h_2) & (\int_{t-h_2}^{t} x(t)dt)^T \end{bmatrix} \tag{26}$$

From (24) and (25), we have

$$\dot{V}_5(t) \leq \xi^T(t)\Psi\xi(t) + \bar{h}_1^2\dot{x}^T(t)S_{22}\dot{x}(t) \tag{27}$$

where

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & -S_{12}^T & \bar{h}_1^2S_{12}C & 0 & \bar{h}_1^2S_{12}A_2 \\ * & -S_{22} & S_{12}^T & 0 & 0 & 0 \\ * & * & -S_{11} & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix},$$

$$\begin{aligned}
\Psi_{11} &= \bar{h}_1^2(S_{11} + S_{12}A + A^TS_{12}^T) - S_{22}, \\
\Psi_{12} &= \bar{h}_1^2S_{12}A_1 + S_{22},
\end{aligned}$$

We also have

$$\begin{aligned}
\dot{V}_6(t) &= 2\xi^T(t)\begin{bmatrix} H \\ 0_{3\times 2}\end{bmatrix}\tilde{P}\begin{bmatrix} A & A_1 & 0_{1\times 3} & A_2 \\ I & -I & 0_{1\times 3} & 0\end{bmatrix}\xi(t) \\
&= \xi^T(t)\Upsilon\xi(t) \qquad (28)
\end{aligned}$$

where

$$\begin{aligned}
\Upsilon &= \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & 0 & 0 & P_{11}A_2 \\ * & \Upsilon_{22} & \Upsilon_{23} & 0 & 0 & -C^TP_{11}A_2 \\ * & * & 0 & 0 & 0 & P_{12}^TA_2 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \\
\Upsilon_{11} &= P_{12}+P_{12}^T+P_{11}A+A^TP_{11}^T, \\
\Upsilon_{12} &= P_{11}A_1-P_{12}-A^TP_{11}C-P_{12}^TC, \\
\Upsilon_{13} &= A^TP_{12}+P_{22}, \Upsilon_{23}=A_1^TP_{12}-P_{22}, \\
\Upsilon_{22} &= P_{12}^TC+C^TP_{12}-A_1^TP_{11}C-C^TP_{11}A_1.
\end{aligned}$$

Using similar process for deriving (24) and (25), we have

$$\dot{V}_7(t) \le \xi^T(t)\digamma\xi(t)+h_2^2\dot{x}^T(t)Q_{22}\dot{x}(t) \qquad (29)$$

where

$$\begin{aligned}
\digamma &= \begin{bmatrix} \digamma_{11} & \digamma_{12} & 0 & h_2^2Q_{12}C & Q_{22} & \digamma_{16} \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & -Q_{22} & Q_{12}^T \\ * & * & * & * & * & -Q_{11} \end{bmatrix} \\
\digamma_{11} &= h_2^2(Q_{11}+Q_{12}A+A^TQ_{12}^T)-Q_{22}, \\
\digamma_{16} &= h_2^2Q_{12}A_2-Q_{12}^T, \digamma_{12}=h_2^2Q_{12}A_1.
\end{aligned}$$

$$\dot{V}_8(t)=\xi^T(t)\bar{\Sigma}\xi(t) \qquad (30)$$

where

$$\begin{aligned}
\bar{\Sigma} &= \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & 0 & 0 & -Z_{12} & \bar{\Sigma}_{16} \\ * & \bar{\Sigma}_{22} & 0 & 0 & C^TZ_{12} & \bar{\Sigma}_{26} \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & -Z_{22} \\ * & * & * & * & * & \bar{\Sigma}_{66} \end{bmatrix} \\
\bar{\Sigma}_{11} &= Z_{12}+Z_{12}^T+Z_{11}A+A^TZ_{11}^T, \\
\bar{\Sigma}_{12} &= Z_{11}A_1-A^TZ_{11}C-Z_{12}^TC, \\
\bar{\Sigma}_{16} &= Z_{11}A_2+A^TZ_{12}+Z_{22}, \\
\bar{\Sigma}_{26} &= A_1^TZ_{12}-C^TZ_{11}A_2, \\
\bar{\Sigma}_{22} &= -C^TZ_{11}A_1-A_1^TZ_{11}C, \\
\bar{\Sigma}_{66} &= Z_{12}^TA_2+A_2^TZ_{12}.
\end{aligned}$$

Considering (20)–(30) together, we have

$$\dot{V}(t)=\sum_{k=1}^{8}\dot{V}_k(t)\le\xi^T(t)[\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T]\xi(t) \qquad (31)$$

where $\Sigma_{ij}(i,j=1,2)$ is defined in Theorem 1.

By the Schur complement, (10) implies that $\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T<0$. Therefore, from (31), we have $\dot{V}(t)<0$ for any $\xi^T(t)\neq 0$. This implies that system (1)-(3) is asymptotically stable if the LMIs (10) is feasible. This completes the proof. □

Remark 1: The augmented matrix (26) is constructed differently from those in the existing methods. No correlated item exists in $\xi(t)$. Therefore, it is not necessary to introduce free weighting matrices to reveal the relationships among the correlated items.

Remark 2: It is worth mentioning that the terms Q_{12}, P_{12}, S_{12} and Z_{12} in (16)- (19) are used to reduce the conservativeness. The cross terms, such as $2\int_{-h_1}^{0}\int_{t+s}^{t}\dot{x}^T(t)S_{12}x(t)dtds$, are introduced into the Lyapunov-Krasovskii functional in our method and are selected in accordance with (9). Therefore, the additional design matrix gives a potential relaxation [22], and consequently less conservative results can be expected. This will be be demonstrated in Section 4 through examples.

Now we are in the position to consider neutral systems with norm-bounded uncertainties. In terms of (6), system (1)-(3) can be written as

$$\begin{aligned}
\dot{x}(t) &= Ax(t)+A_1x(t-h_1)+C\dot{x}(t-h_1) \\
&\quad +A_2\int_{t-h_2}^{t}x(s)ds+MF(t)y \\
y &= E_0x(t)+E_1x(t-h_1) \\
&\quad +E_c\dot{x}(t-h_1)+E_2\int_{t-h_2}^{t}x(s)ds \\
x(t) &= \phi(t), t\in[-\max(h_1,h_2),0] \qquad (32)
\end{aligned}$$

According to $F^T(t)F(t)\le I$, it follows that

$$[F(t)y]^T[F(t)y]\le y^Ty \qquad (33)$$

Similar to Theorem 1, the following results are given.

Theorem 2: Given scalars $\bar{h}_1\ge 0$ and $h_2\ge 0$, the neutral system described by (1)-(3) is asymptotically stable for any $0\le h_1\le\bar{h}_1$, if there exist matrices $R_i(i=1,\ldots,4)$, $\tilde{Q}>0$, $\tilde{P}>0,\tilde{R}>0$ and $\tilde{Z}>0$, any matrices, a scalar $\varepsilon\ge 0$ such that the following LMI hold

$$\begin{bmatrix} \Sigma_{11} & \Omega_{11} & \Sigma_{12} & \Omega_{12} \\ * & -\varepsilon I & \Omega_{13} & 0 \\ * & * & \Sigma_{22} & 0 \\ * & * & * & -\varepsilon I \end{bmatrix}<0 \qquad (34)$$

where Σ_{11}, Σ_{12},Σ_{22} is defined in Theorem 1 and

$$\begin{aligned}
\Omega_{11} &= \begin{bmatrix} \Lambda_1 & 0 & P_{12} & -(P_{11}+Z_{11})C & 0 & Z_{12}\end{bmatrix}^TM, \\
\Omega_{12} &= \begin{bmatrix} \varepsilon E_0 & \varepsilon E_1 & 0 & \varepsilon E_c & 0 & \varepsilon E_2\end{bmatrix}^T, \\
\Omega_{13} &= \begin{bmatrix} M^TR_1 & \bar{h}_1M^TS_{22} & h_2M^TQ_{22} & \Lambda_2\end{bmatrix}, \\
\Lambda_1 &= R_0+Z_{11}+P_{11}+\bar{h}_1^2S_{12}^T+h_2^2Q_{12}^T, \\
\Lambda_2 &= (\bar{h}_1-h_2)M^TS_{22}.
\end{aligned}$$

Proof: Construct the same Lyapunov candidate functional as (12)-(19), in terms of the described uncertain delay system (32). Using similar process of deriving (20)-(30), we have

$$\dot{V}(t)\le\varrho_1^T(t)\left\{\begin{bmatrix} \Sigma_{11} & \Omega_{11} \\ * & 0\end{bmatrix}-\varphi\Sigma_{22}^{-1}\varphi^T\right\}\varrho_1(t) \qquad (35)$$

where $\varrho_1^T(t)$=$\left[\begin{array}{cc} \xi^T(t) & [F(t)y]^T \end{array}\right]$, $\varphi = \left[\begin{array}{c} \Sigma_{12} \\ \Omega_{13} \end{array}\right]$ and Σ_{11}, Σ_{12}, Σ_{22} is defined in Theorem 1 and Ω_{11}, Ω_{13} is defined in Theorem 2 respectively. By Schur complement, (34) implies that

$$\varrho_1^T(t)\left\{\left[\begin{array}{cc} \Sigma_{11} & \Omega_{11} \\ * & -\varepsilon I \end{array}\right] - \varphi\Sigma_{22}^{-1}\varphi^T\right\}\varrho_1(t) + \varepsilon y^T y < 0 \quad (36)$$

From (33), it is seen that there exists a scalar $\varepsilon \geq 0$ such that

$$\varepsilon y^T y - \varepsilon[F(t)y]^T[F(t)y] \geq 0 \quad (37)$$

Then combining (35)- (37), we have $\dot{V}(t) < 0$ for any $\varrho_1^T(t) \neq 0$. Therefore, system (32) is asymptotically stable if the LMIs (34) is feasible. This completes the proof. □

Remark 3: Compared with Moon's inequalities [11], Lemma 1 is a more general and tighter bounding technology for dealing with cross terms. Therefore, the results derived here are expected to be less conservative. Moreover, the item $[F(t)y]$ in $\varrho_1(t)$ brings us convenience for dealing with time-varying norm bound uncertainties.

IV. NUMERICAL EXAMPLES

This section aims to demonstrate the effectiveness of the proposed approach. For comparisons with the existing methods [23], [7], [3], we choose the same systems used in these references.

A. Example 1 (Chen, Lien, Fan and Chou [23], Yue, Won and Kwon [7], Chen and Zheng [3])

Consider the following system with discrete and distributed delays

$$\begin{aligned}\dot{x}(t) &= -\left[\begin{array}{cc} 0.9 & 0 \\ 0 & 0.9 \end{array}\right]x(t) + \left[\begin{array}{cc} -1 & -0.12 \\ 0.12 & -1 \end{array}\right]x(t-h_1) \\ &\quad + \left[\begin{array}{cc} -0.2 & 0 \\ 0.2 & -0.1 \end{array}\right]\int_{t-h_2}^{t} x(s)ds \quad (38)\end{aligned}$$

The results on the maximum allowable delay bound h_1 are tabulated in Table I for $h_2 = 1$.

TABLE I

THE MAXIMUM ALLOWABLE DELAY BOUND h_1 WHEN h_2=1.

Method	CLFC [23]	YWK [7]	CZ [3]	Theorem 1
h_1	0.9086	1.8302	2.8011	3.1668
Improvement	$\geq 248\%$	$\geq 73\%$	$\geq 13\%$	-

Using the criterion in this paper, the maximum value of h_1 for the nominal system to be asymptotically stable is h_1 $\leq$3.1668. By the criteria in [23], [7], and [3], the system (38) is asymptotically stable for any h_1 satisfying h_1 $\leq$0.9086, h_1 $\leq$1.8302, and h_1 $\leq$2.8011, respectively. Our results have improved those results significantly (248%, 73%, and 13%, respectively). The results given in Table I indicate the delay-dependent stability in this paper are much less conservative than existing ones reported in the open literature.

B. Example 2 (Han [5], He, Wu, She and Liu [12], Xu, Lam and Zou [24])

Consider the following uncertain neural delay-differential system

$$\dot{x}(t) - C\dot{x}(t-h_1) = [A + \Delta A(t)]\,x(t) + [B + \Delta B(t)]\,x(t-h_1) \quad (39)$$

where

$$\begin{aligned} A &= \left[\begin{array}{cc} -0.9 & 0.2 \\ 0.1 & -0.9 \end{array}\right], B = \left[\begin{array}{cc} -1.1 & -0.2 \\ -0.1 & -1.1 \end{array}\right], \\ C &= \left[\begin{array}{cc} -0.2 & 0 \\ 0.2 & -0.1 \end{array}\right]. \end{aligned}$$

The uncertain matrices $\Delta A(t)$ and $\Delta A_d(t)$ satisfy $\|\Delta A(t)\| \leq \lambda$, $\|\Delta A_d(t)\| \leq \lambda$, $\lambda > 0$.

$$M = diag\{\lambda, \lambda\}, E_a = E_d = I. \quad (40)$$

For different values of λ, Table II lists the maximum allowable delay bounds from Han [5], He, Wu, She and Liu [12], Xu, Lam and Zou [24] and Theorem 2 of this paper. Similar to Example 1, this example also shows that the stability criterion derived in this paper gives much less conservative results than those in [5], [12], [24]. It can be seen from Table II that the performance improvement over Xu, Lam and Zou [24] is significant.

V. CONCLUSION

An innovative approach has been proposed to significantly improve the performance for the delay-dependent robust stability of uncertainty neutral systems with discrete and distributed delays. With the proposed approach, improved delay-dependent stability conditions for such systems have been developed, which are expressed in LMIs. Neither model transformation nor free weighting matrices are employed in deriving the delay-dependent results. The innovative approach and performance improvement result from the well chosen Lyapunov-Krasovskii functional and a tighter bound technology for cross terms. Numerical examples have demonstrated that the proposed conditions are much less conservative than the existing ones reported in the open literature.

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REFERENCES

[1] Q. L. Han. Robust stability of uncertain delay-differential systems of neutral type. *Automatica*, 38:719–723, 2002.

[2] C. H. Lien and J. D. chen. Discrete-delay-independent and discrete-delay-dependent criteria for a class of neutral systems. *J. Dynamic Systems, Measurement, and Control*, 136:177–184, 2001.

TABLE II

THE MAXIMUM ALLOWABLE DELAY BOUNDS FOR EXAMPLE 2.

λ	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
Han [5]	1.61	1.51	1.41	1.30	1.19	1.08	0.96	0.83
HWXL [12]	1.6527	1.5353	1.4276	1.3282	1.2361	1.1502	1.0699	0.9945
XLZ [24]	1.7220	1.6032	1.4935	1.3919	1.2978	1.2103	1.1288	1.0526
Theorem 2	1.7890	1.6805	1.5801	1.4868	1.4000	1.3190	1.2433	1.1725
Imp. over [24]	$\geq 3.8\%$	$\geq 4.8\%$	$\geq 5.7\%$	$\geq 6.8\%$	$\geq 7.8\%$	$\geq 8.9\%$	$\geq 10.1\%$	$\geq 11.3\%$

[3] W. H. Chen and W. X. Zheng. Delay-dependent robust stabilization for uncertain neutral systems with destributed delays. *Automatica*, 43:95–104, 2007.

[4] K. Gu. An improved stability criterion for systems with distributed delays. *International Journal of Control*, 13:819–831, 2003.

[5] Q. L. Han. A descriptor system approach to robust stability of uncertain neutral systems with discrete and distributed delays. *Automatica*, 40:1791–1796, 2004.

[6] L. Xie, E. Fridman, and U. Shaked. Robust control of distributed delay systems with applications to combustion control. *IEEE Transactions on Automatic Control*, 46:1930–1935, 2001.

[7] D. Yue, S. Won, and O. Kwon. Delay dependent stability of neutral systems with time delay: an lmi approach. *IEE Proceedings: Control Theory and Applications*, 150(1):23–27, 2003.

[8] F. Zheng and P. M. Frank. Robust control of uncertain distributed delay systems with applications to the stabilizaiton of combustion in rocket motor chambers. *Automatica*, 38:487–497, 2002.

[9] E. Fridam. New lyapunov-krasovskii functions for stability of linear retarded and neutral types systems. *Systems & Control Letters*, 43:309–319, 2001.

[10] Park J. H. A new delay-dependent criterion for neutral systems with multiple delays. *Journal of Computational and Applied Mathematics*, 136:177–184, 2001.

[11] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee. Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*, 74:1447–1455, 2001.

[12] Y. He, M. Wu, J.-H. She, and G.-P. Liu. Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *Systems & Control Letters*, 51:57–65, 2004.

[13] C. Peng and D. Yue. State feedback controller design of networked control systems with parameter uncertainty and state delay. *Asia Journal of Control*, 8(4):385–392, 2006.

[14] M. Wu, Y. He, and J.-H. She. New delay-dependent stability criteria and stabilizing method for neutral systems. *IEEE Transactions on Automatic Control*, 49:2266–2271, 2004.

[15] S. Xu, J. Lam, and Y. Zou. New results on delay-depnedent robust H_∞ control for systems with time-varying delay. *Automatica*, 42:343–348, 2006.

[16] D. Yue, Q. L. Han, and J. Lam. Network-based robust H_∞ control of systems with uncertainty. *Automatica*, 41:999–1007, 2005.

[17] E. Fridman and U. Shaked. A descriptor system approach to H_∞ control of linear time-delay systems. *IEEE Transactions on Automatic Control*, 47:253–270, 2002.

[18] K. Gu, V.L. Kharitonov, and J. Chen. *Stability of time-delay systems*. Birkhauser, 2003.

[19] Y. He, Q. G. Wang, C. Lin, and M. Wu. Augmented lyapunov fuctional and delay-dependent stability criteria for neutral systems. *International Journal of Robust and Nonlinear Control*, 15:923–933, 2005.

[20] M .N. A. Parlakçı. Improved robust stability criteria and design of robust stablizing controller for uncertain linear time-delay systems. *International Journal of Robust and Nonlinear Control*, 16:pp.599–636, 2006.

[21] Chen Peng and Yu-Chu Tian. Delay-dependent robust stability criteria for uncertain systems with interval time-varying delay. *Journal of Computational and Applied Mathematics*, page doi:10.1016/j.cam.2007.03.009, 2007.

[22] M. N. A. Parlakci. Robust stability of uncertain time-varying state-delayed ststems. *IEE Proceedings: Control Theory and Applications*, 153(4):469–477, 2006.

[23] J. D. Chen, C. H. Lien, K. K. Fan, and J. H. Chou. Criteria for asymptotic stability of c class of neutral system via a LMI approach. *IEE Proceedings: Control Theory and Applications*, 148(6):442–447, 2001.

[24] S. Xu, J. Lam, and Y. Zou. Further results on delay-dependent robust stability conditions of uncertain neutral systems. *International Journal of Robust and Nonlinear Control*, 15:233–246, 2005.